As was mentioned earlier, the stellar photosphere, the "surface of a star," is defined as the layer of its atmosphere at an optical depth $\tau_\lambda = 2/3$ for a given wavelength λ. But why exactly 2/3? Following Carroll & Ostlie (2006), assuming that the optical depth τ_λ represents the number of photons' mean free paths to the surface, we should see $\tau_\lambda \approx 1$ into the atmosphere of a star. Those photons originating at an optically thick deeper regions at $\tau_\lambda \gg 1$ will not be able to reach the surface without being absorbed or scattered. However, why not define the surface at $\tau_\lambda = 0$?

The surface ($r = R$) flux of a perfect blackbody, according to the Stefan–Boltzmann equation $L = 4\pi R^2 \sigma T_e^4$ and the inverse square law $F = L/4\pi r^2$, depends on the effective temperature:

$$F_{surf} = \sigma T_e^4. \tag{1}$$

Assuming we treat stars as blackbody emitters, their surfaces can, thus, be defined as layers of their atmosphere where $T = T_e$, with T_e coming from, for example, spectral observations. In other words, photons have to originate at this photospheric region which is, in principle, the origin of the star's continuum emission.

In order to find the optical depth of this layer of star's atmosphere, we need to understand how T varies with τ_λ. For this, we make a number of assumptions. First, we assume that the atmosphere is in the state of local thermodynamic equilibrium (LTE), that is the temperature in the atmosphere does not change *significantly* with height. This leads to the equality of radiative and surface fluxes: $F_{rad} = F_{surf} = \sigma T_e^4$.

During the consideration of the flow of radiation in a stellar atmosphere, it is, then, convenient to assume that it is a flat (plane-parallel) slab. This comes from the fact that the curvature radius of a star's atmosphere significantly exceeds its thickness. The third simplification comes from the assumption that the atmosphere is *grey*, that is, its opacity is wavelength-independent and that atmospheric levels can be determined by unique values of τ_v.

One method to solve the equation of radiative transfer is the Eddington approximation. It assumes that the radiation field is comprised of two streams—ingoing and outgoing—each of which is isotropic over half the hemisphere (Huang 1968). According to it, it can be shown that the mean intensity of light in a plane-parallel grey atmosphere depends on the F_{rad} and τ_v as

$$\frac{4\pi}{3}\langle I \rangle = F_{rad}\left(\tau_v + \frac{2}{3}\right). \tag{2}$$

The intensity of light $\langle I \rangle$ tends to become equal to the local value of the source function, which, in its turn, in the presence of LTE, equals to the Planck function that describes the blackbody radiation curve (integrated over all wavelengths, it is equal to $\sigma T^4/\pi$). Thus, it can be shown that the temperature of a plane-parallel grey atmosphere in LTE, assuming the Eddington approximation, depends on the optical depth τ_v and the effective temperature T_e as

$$T^4 = \frac{3}{4} T_e^4 \left(\tau_v + \frac{2}{3}\right) \tag{3}$$

At $\tau_v = 2/3$, $T = T_e$. Assuming we treat stars as blackbodies then it follows that their surfaces, from the Stefan–Boltzmann definition of the surface of a blackbody via T_e, lie at an optical depth of $\tau_v = 2/3$. Thus, that layer of stellar atmosphere at a depth $\tau_\lambda = 2/3$ can be thought of a region where the continuum emission originates. The definition of this depth is intrinsically connected to the Stefan–Boltzmann equation and the Planck function. The meaning of this is that the surface of a star, its photosphere, is the origin of photons that build one of the primary features of it as a blackbody emitter—its continuum spectrum.

REFERENCES

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