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ILT 1 WEEKS 5–6, VERSION 3

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1. INTRODUCTION

Following (Zwicky 1937) we start with the virial theorem that relates the average kinetic energy $\langle K\rangle$ to the average gravitational potential energy $\langle U\rangle$ of bodies in a system (cluster, galaxy, etc) in the state of equilibrium: $\langle K\rangle = -\frac{1}{2}\langle U\rangle$, with brackets denoting an average value. We can measure radial velocities V_r of individual bodies in a system (of stars in a globular cluster or a galaxy, of galaxies in a cluster of galaxies) via spectroscopic means. The average of those measurements $\langle V_r\rangle$ would represent the systemic velocity – of the system as a whole as it can be moving from or towards the observer. Thus, radial velocity of an individual body of a system relative to the system's center of mass is given by $v_r^2 = (V_r - \langle V_r\rangle)^2$. In other words, in order to determine intrinsic radial velocities of individual bodies of a system we shall remove systemic velocity of a system—the average—from individual measurements.

The standard deviation[2] of radial velocity measurements of a system composed of N bodies is given by

$$\sigma_r \equiv \sqrt{\frac{1}{N}\sum_{i=1}^{N}(V_{r,i} - \langle V_r\rangle)^2}. \tag{1}$$

And since $v_r^2 = (V_r - \langle V_r\rangle)^2$, we have a relationship between the average of squared intrinsic radial velocities of bodies in a system and the square of standard deviation of the radial velocity measurements:

$$\langle v_r^2\rangle = \sigma_r^2. \tag{2}$$

Assuming the whole system is isotropic (velocities of its bodies are the same in all directions) average intrinsic velocity of bodies in a system $\langle v^2\rangle = \langle v_r^2\rangle + \langle v_\theta^2\rangle + \langle v_\phi^2\rangle = 3\langle v_r^2\rangle$, whereby (r, θ, ϕ) denote velocity components in a three-dimensional spherical coordinate system (Carroll & Ostlie 2006). Using equation 2 here, we derive average intrinsic velocity of bodies in a system from standard deviation (dispersion) of radial velocity measurements:

$$\langle v^2\rangle = 3\sigma_r^2. \tag{3}$$

The average kinetic energy of bodies in a system is $\langle K\rangle = 1/2\langle m\rangle\langle v^2\rangle = 1/2\langle m\rangle(3\sigma_r^2)$ while their gravitational potential energy is $\langle U\rangle = -(GM\langle m\rangle/\langle r\rangle)$, where $\langle r\rangle$ is the average distance of a body from the center of mass M in a system. Above, $\langle m\rangle$ denotes average mass of a body. Assuming average body is located at half the radius R of the whole system—which is an acceptable approximation for certain systems like globular clusters—we have $\langle r\rangle \sim R/2$. Now, using the virial theorem and energies defined above:

$$\frac{1}{2}\langle m\rangle(3\sigma_r^2) \sim \frac{1}{2}\frac{GM\langle m\rangle}{R/2} \tag{4}$$

With $\langle m\rangle$ at both sides cancelled and rearranging the terms:

$$M \sim \frac{3}{2}\frac{R\sigma_r^2}{G} \tag{5}$$

2. ESTIMATING THE MASS OF THE COMA CLUSTER (ABELL 1656)

It is interesting that my equation 5 differs from Zwicky's equation 33. I could not find an explanation for this. Checking input with Zwicky's formula and data first, I got 8.51×10^{46} g estimate for the mass of the Coma cluster. This is consistent with his number of 9×10^{46} g. For the input, I have used $3\sigma_r^2 = 1.5 \times 10^{16}$ cm^2 s^{-2} and $R = 1.89211 \times 10^{24}$ cm from his paper. In solar masses, Zwicky's result is $4.28 \times 10^{13} M_\odot$.

With new radial velocity data from Biviano et al. (1995), I have found new value of $3\sigma_r^2 = 4.73838 \times 10^{16}$ cm^2 s^{-2} with eight distinct velocity data outliers removed from the set. This is more than double the value Zwicky used. The velocity dispersion of the cluster is $\sim$1257 km s^{-1} while systemic velocity is $\sim$7357 km s^{-1}. Based on the new radial velocity data, Zwicky's Abell 1656 radius estimate and equation 5, I have found the mass of the cluster to be:

$$M_{cl} \sim \frac{1.89211 \times 10^{24}\ \text{cm} \times 4.73838 \times 10^{16}\ \text{cm}^2\ \text{s}^{-2}}{2 \times 6.67259 \times 10^{-8}\ \text{cm}^3\ \text{g}^{-1}\ \text{s}^{-2}}$$
$$\sim 1.50972 \times 10^{48}\ \text{g}/1.99 \times 10^{33}\ \text{g}/M_\odot \sim 3.38 \times 10^{14} M_\odot \tag{6}$$

REFERENCES

Biviano, A., Durret, F., Gerbal, D., et al. 1995, A&AS, 111, 265
Carroll, B. W., & Ostlie, D. A. 2006, An introduction to modern astrophysics and cosmology
Zwicky, F. 1937, ApJ, 86, 217

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[2] Here, the standard deviation is defined without the Bessel's correction $1/N-1$. The correction would remove bias because the true mean (systemic velocity) is not known. The residual vector after removing systemic velocities is $(V_{r,1} - \langle V_r\rangle, \ldots, V_{r,N} - \langle V_r\rangle)$. While there are N independent samples, there are only $N - 1$ independent residuals, as they sum to 0. On the other hand, $\langle v_r^2\rangle \equiv \frac{1}{N}\sum_{i=1}^{N}(V_{r,i} - \langle V_r\rangle)^2$ which is identical to the square of the uncorrected standard deviation of V_r. The simplicity of this passage leads to equation 2.