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ILT 1 WEEKS 9–10

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1. JUPITER

I would like to discuss Jupiter's energy balance, in particular the excess heat radiated by it to space and the amount of Kevin-Helmholtz contraction required to support our measurements. Let's start with Jupiter's measured effective temperature $T_{measured} = 134$ K (Aumann et al. 1969). At the present time, the Sun emits energy at the rate of $Q = 3.87 \times 10^{26}$ W. Jupiter is at the distance $r = 7.78 \times 10^{11}$ m from the Sun, so the flux of the solar energy per unit area of Jupiter is

$$S_0 = \frac{Q}{4\pi r^2}. \tag{1}$$

For Jupiter, $S_0 = 50.88$ W m^{-2}. With Jupiter's radius $a = 6.99 \times 10^7$ m, total solar radiation incident on it would be

$$Q_{incident} = S_0 \pi a^2. \tag{2}$$

That is, $Q_{incident} = 7.81 \times 10^{17}$ W. Assuming average Jupiter's albedo $\alpha = 0.52$ and that some part of the solar energy is reflected back to space and does not participate in the planet's energy budget, total solar radiation absorbed by Jupiter would be

$$Q_{absorbed} = (1 - \alpha) Q_{incident}. \tag{3}$$

In the case of Jupiter, this value is 3.75×10^{17} W (Zong-Liang 2013). On the other hand, in order for the planet to have an observed thermal emission at $T_{measured} = 134$ K, according to the Stefan-Boltzmann law, it must radiate with power $S_r = \sigma T^4_{measured}$, where σ is Stefan-Boltzmann constant, 5.67×10^{-8} W m^{-2} K^{-4}. For the whole of Jupiter, the energy radiated equals to

$$Q_{radiated} = 4\pi a^2 S_r. \tag{4}$$

Thus, the 134 K temperature means that Jupiter is radiating 1.12×10^{18} W of energy into space, however it absorbs only 3.75×10^{17} W from the Sun, as was shown earlier. This is 2.99 times more energy radiated than absorbed. In absolute terms, this excess energy is $Q_{excess} = Q_{radiated} - Q_{absorbed} = 7.48 \times 10^{17}$ W.

Now let us consider the case of Kevin-Helmholtz contraction. Gravitational potential energy of a body is

$$U = -\frac{GM^2}{a}, \tag{5}$$

where G is the gravitational constant and M is the mass of the planet. As a (planet's radius) decreases, U increases (becomes more negative). Assuming excess energy comes from and only the contraction mechanism:

$$Q_{excess} = \frac{GM^2}{a^2}\frac{da}{dt}, \tag{6}$$

so rate of change in planetary radius a

$$\frac{da}{dt} = \frac{Q_{excess} a^2}{GM^2}. \tag{7}$$

With Jupiter's mass $M = 1.90 \times 10^{27}$ kg, $\frac{da}{dt} = 0.5$ mm/yr that seems plausible over astronomically long time scales (Gary ????).

2. OTHER PLANETS

Using equations above, I have derived $\frac{da}{dt}$ for other planets showing excess radiation. For Saturn with $T_{measured} = 97$ K (Aumann et al. 1969) $\frac{Q_{radiated}}{Q_{absorbed}} = 2.53$ and $\frac{da}{dt} = 0.64$ mm/yr. For Neptune with $T_{measured} = 55$ K (Loewenstein et al. 1977) $\frac{Q_{radiated}}{Q_{absorbed}} = 2.32$ and $\frac{da}{dt} = 0.06$ mm/yr. For Uranus with $T_{measured} = 59$ K I derived $\frac{Q_{radiated}}{Q_{absorbed}} = 1.06$ which is identical to results obtained by Pearl et al. (1990) and $\frac{da}{dt} = 0.01$ mm/yr.

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