

ILT 1 WEEKS 5–6, VERSION 2

M. USATOV¹

1. INTRODUCTION

Following (Zwicky 1937) we start with the virial theorem that relates the average kinetic energy $\langle K \rangle$ to the average gravitational potential energy $\langle U \rangle$ of bodies in a system (cluster, galaxy, etc) in the state of equilibrium: $\langle K \rangle = -\frac{1}{2}\langle U \rangle$, with brackets denoting an average value. We can measure radial velocities V_r of individual bodies in a system (of stars in a globular cluster or a galaxy, of galaxies in a cluster of galaxies) via spectroscopic means. The average of those measurements $\langle V_r \rangle$ would represent the systemic velocity – of the system as a whole as it can be moving from or towards the observer. Thus, radial velocity of an individual body of a system relative to the system’s center of mass is given by $v_r = V_r - \langle V_r \rangle$. In other words, in order to determine intrinsic radial velocities of individual bodies of a system we shall remove systemic velocity of a system—the average—from individual measurements.

The standard deviation of radial velocity measurements of a system composed of N bodies is given by

$$\sigma_r \equiv \sqrt{\frac{1}{N} \sum_{i=1}^N (V_{r,i} - \langle V_r \rangle)^2}. \quad (1)$$

And since $v_r^2 = (V_r - \langle V_r \rangle)^2$, we have a relationship between the average of squared intrinsic radial velocities of bodies in a system and the square of standard deviation of the radial velocity measurements:

$$\langle v_r^2 \rangle = \sigma_r^2. \quad (2)$$

Assuming the whole system is isotropic (velocities of its bodies are the same in all directions) average intrinsic velocity of bodies in a system $\langle v^2 \rangle = \langle v_r^2 \rangle + \langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle = 3\langle v_r^2 \rangle$, whereby (r, θ, ϕ) denote velocity components in a three-dimensional spherical coordinate system (Carroll & Ostlie 2006). Using equation 2 here, we derive average intrinsic velocity of bodies in a system from standard deviation (dispersion) of radial velocity measurements:

$$\langle v^2 \rangle = 3\sigma_r^2. \quad (3)$$

The average kinetic energy of bodies in a system is $\langle K \rangle = 1/2\langle m \rangle \langle v^2 \rangle = 1/2\langle m \rangle (3\sigma_r^2)$ while their gravitational potential energy is $\langle U \rangle = -(GM\langle m \rangle / \langle r \rangle)$, where $\langle r \rangle$ is the average distance of a body from the center of mass M in a system. Above, $\langle m \rangle$ denotes average mass

of a body. Assuming average body is located at half the radius R of the whole system—which is an acceptable approximation for certain systems like globular clusters—we have $\langle r \rangle \sim R/2$. Now, using the virial theorem and energies defined above:

$$\frac{1}{2}\langle m \rangle (3\sigma_r^2) \sim \frac{1}{2} \frac{GM\langle m \rangle}{R/2} \quad (4)$$

With $\langle m \rangle$ at both sides cancelled and rearranging the terms:

$$M = \frac{3}{2} \frac{R\sigma_r^2}{G} \quad (5)$$

2. ESTIMATING THE MASS OF THE COMA CLUSTER
 (ABELL 1656)

It is interesting that my equation 5 differs from Zwicky’s equation 33. I could not find an explanation for this. Checking input with Zwicky’s formula and data first, I got 8.51×10^{46} g estimate for the mass of the Coma cluster. This is consistent with his number of 9×10^{46} g. For the input, I have used $3\sigma_r^2 = 1.5 \times 10^{16}$ cm² s⁻² and $R = 1.89211 \times 10^{24}$ cm from his paper. In solar masses, Zwicky’s result is $4.28 \times 10^{13} M_\odot$.

With new radial velocity data from Biviano et al. (1995), I have found new value of $3\sigma_r^2 = 3.5494 \times 10^{16}$ cm² s⁻² with four distinct velocity data outliers removed from the set. This is double the value Zwicky used. The velocity dispersion of the cluster is ~ 1087 km s⁻¹ while systemic velocity is ~ 6791 km s⁻¹. Based on the new radial velocity data, Zwicky’s Abell 1656 radius estimate and equation 5, I have found the mass of the cluster to be $\sim 7.59 \times 10^{14} M_\odot$:

$$M_{cl} \sim \frac{3 \times 1.89211 \times 10^{24} \text{ cm} \times 3.5494 \times 10^{16} \text{ cm}^2 \text{ s}^{-2}}{2 \times 6.67259 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}} \\ \sim 1.50972 \times 10^{48} \text{ g} / 1.99 \times 10^{33} \text{ g} / M_\odot \sim 7.59 \times 10^{14} M_\odot \quad (6)$$

REFERENCES

- Biviano, A., Durret, F., Gerbal, D., et al. 1995, A&AS, 111, 265
 Carroll, B. W., & Ostlie, D. A. 2006, An introduction to modern astrophysics and cosmology
 Zwicky, F. 1937, ApJ, 86, 217

¹ maxim.usatov@bcsatellite.net